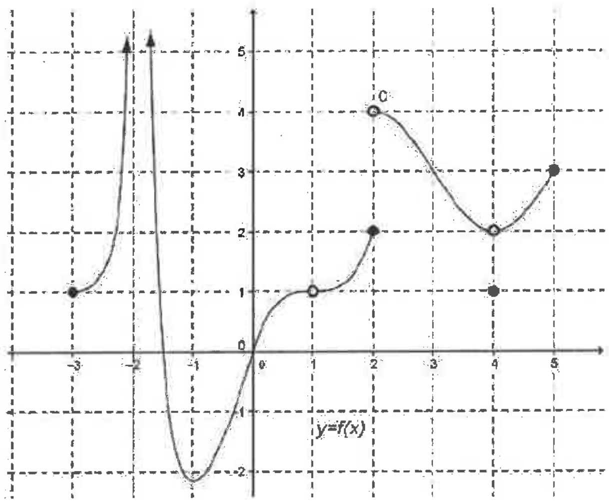
When is a function continuous?

Recall: A function, $f(x)$, is continuous at $x = c$ if:

I. $f(c)$ is defined

II. $\lim_{x \to c} f(x)$ exists

III. $f(c) = \lim_{x \to c} f(x)$



Given the graph of a function, $f(x)$, above, determine if $f(x)$ is continuous at the indicated *x*-value.

1) $x = -2$ $f(-2) = \varnothing$ $f(x)$ is not cont @ $x = -2$.	2) $x = 0$ $f(0) = 0$ $\lim_{x \to 0} f(x) = 0$ $f(0) = \lim_{x \to 0} f(x)$ $f(x)$ is cont. @ $x = 0$
3) $x = 1$ $f(1) = \varnothing$ $f(x)$ is not cont @ $x = 1$.	4) $x = 2$ $\lim_{x \to 2^-} f(x) = 2 \neq \lim_{x \to 2^+} f(x) = 4$ $\lim_{x \to 2} f(x)$ DNE $f(x)$ is not cont. @ $x = 2$.
5) $x = 4$ $f(4) = 1 \neq \lim_{x \to 4} f(x) = 2$ $f(x)$ is not cont. @ $x = 4$	

6) Determine the domain of $f(x)$.

$[-3,-2)\cup(-2,1)\cup(1,5]$

7) Determine the values of c for which the $\lim\limits_{x\to c} f(x)$ exists.

$[-3,-2)\cup(-2,2)\cup(2,5]$

8) Determine the interval on which $f(x)$ is continuous.

$[-3,-2)\cup(-2,1)\cup(1,2)\cup$
$(2,4)\cup(4,5]$

Part II: Verify if the function $f(x)$ is continuous at the given value of x. Show all three steps of verification.

1. $f(x)=x^2;\ x=2$

$f(2)=4 = \lim\limits_{x\to 2} f(x)=4$

$f(x)$ is cont @ $x=2$.

2. $f(x)=\begin{cases} 3x^2-4x, & x<1 \\ x-2, & x\ge 1 \end{cases};\quad x=1$

I. $f(1)=-1$

II $\lim\limits_{x\to 1^-}(3x^2-4x)=-1 = \lim\limits_{x\to 1^+}(x-2)=-1$

$\lim\limits_{x\to 1} f(x)=-1$

III. $f(1)=\lim\limits_{x\to 1} f(x)$

$f(x)$ is cont. @ $x=1$.

3. $g(x)=\begin{cases} \dfrac{1}{x+4}, & x\le -1 \\ 3^x, & -1<x<2, \\ x^2-1, & x\ge 2 \end{cases}\quad x=-1$ and $x=2$

$x=-1$

I. $f(-1)=\frac{1}{3}$

II. $\lim\limits_{x\to -1^-}\frac{1}{x+4}=\frac{1}{3}$

$\lim\limits_{x\to -1^+} 3^x=\frac{1}{3}$

$\lim\limits_{x\to -1} f(x)=\frac{1}{3}$

III. $f(-1)=\lim\limits_{x\to -1} f(x)$

$f(x)$ is cont @ $x=-1$.

$x=2$

I. $f(2)=3$

II. $\lim\limits_{x\to 2^-} 3^x=9$

$\lim\limits_{x\to 2^+}(x^2-1)=3$

$\lim\limits_{x\to 2} f(x)$ DNE

$f(x)$ is not cont @ $x=2$.

4. $f(x)=\begin{cases} x-x^2, & x<1 \\ \ln x, & x=1, \\ x, & x>1 \end{cases}\quad x=1$

I. $f(1)=\ln 1=0$

II. $\left.\begin{array}{l} \lim\limits_{x\to 1^-}(x-x^2)=0 \\ \lim\limits_{x\to 1^+} x=1 \end{array}\right\}\ \lim\limits_{x\to 1} f(x)$ DNE

$f(x)$ is not cont. @ $x=1$.